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Microgravity Isolation System Design: A Modern Control Synthesis Framework

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Manned orbiters will require active vibration isolation for acceleration-sensitive microgravity science experiments. Umbilicals are highly desirable for many experiments, but their presence greatly affects isolation-problem complexity; they must be considered in controller synthesis. A general framework is presented for applying extended H_2 synthesis methods to optimal-controller design so as to provide robust microgravity vibration isolation. Particular attention is given to handling the large parametric uncertainties characteristic of umbilical models. The methodology integrates control and state frequency weighting and input and output disturbance-accommodation techniques into the basic H_2 synthesis approach. Various system models needed for design and analysis are also presented. We conclude with a discussion of a practical design philosophy for using the extended H_2 synthesis methods to develop an optimal microgravity vibration-isolation controller. It is recommended that relative positions, relative velocities, and payload accelerations be the states of choice; that the entire controller (observer plus regulator) be designed as a unit rather than in two separate parts; and that disturbance-accommodation and frequency-weighting design filters be chosen that call for increased virtual stiffness at low frequencies, increased mass at intermediate frequencies, and controller turnoff at higher frequencies.

Nomenclature

A = system dynamical matrix
B = system control input matrix
C = system state output matrix
D = control transmission matrix
E = system disturbance input matrix
$\mathcal{E}$ = expected-value operator
f = stochastic disturbance, units of acceleration
g = gravitational constant
J = performance index
K = regulator feedback gain matrix
L = observer gain matrix
O = zero matrix
P = solution of algebraic Riccati equation for regulator feedback gains
Q = solution of algebraic Riccati equation for observer feedback gains
S = power spectral density matrix
s = Laplace variable
t = time variable, s
u = control, current or force
V = covariance or cross-correlation matrix
W = weighting matrix
$\mathcal{W}$ = filter transfer-function matrix
w = Gaussian white-noise disturbance
x = system state
y = system output
z = system output measurement; frequency-weighting pseudostate
δ = Dirac delta function
ξ = noise disturbance-accommodation pseudostate
τ = time constant, s
ω = circular frequency, rad/s

Subscripts

c = control noise
FB = feedback
f = filter for stochastic disturbance
n = output stochastic disturbance (sensor noise)
s = input stochastic disturbance (process noise)
0 = time $t = 0$; sea level (with g)
1 = state-frequency weighting state-space description (with (A, B, C, D, z); input disturbance-accommodation pseudostates (with ξ); process noise covariance (with V); state (or pseudostate) weightings, applied subsequently to any frequency weighting (with W, $\mathcal{W}$)
2 = control-frequency weighting state-space description (with (A, B, C, D, z); output disturbance-accommodation pseudostates (with ξ); synthesis approach using quadratic cost functional (with H); cross correlation (with V); cross weightings (with W)
3 = measurement noise covariance (with V); control weightings (with W, $\mathcal{W}$)

Superscripts

T = transpose
1 = frequency weighted (with $\boldsymbol{x}$, $\boldsymbol{X}$, $\boldsymbol{u}$, or $\boldsymbol{U}$); system augmented by frequency weighting (with other symbols)
2 = system augmented by frequency weighting and input disturbance accommodation
3 = system augmented by state and control frequency weighting and by input and output disturbance accommodation
4 = system augmented by state and control frequency weighting and by input and output disturbance accommodation, and also including control noise as a process noise input

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5 = system augmented by input and output disturbance accommodation but without frequency weighting (for observation purposes), control noise still included

$\frac{1}{2}$ = square root; spectral factorization

-1 = inverse

$*$ = optimum; complex conjugate transpose

Introduction

ALTHOUGH many scientists have planned or conducted materials-processes and fluid-physics experiments designed for a weightless environment, the currently available facilities have proved far from ideal. Evacuated drop towers can provide only a few seconds of weightlessness at levels on the order of $10^{-6}g_0$ (where g_0 is the gravitational acceleration at sea level). Aircraft flying low-gravity parabolic trajectories can extend the time to about 15–20 s, and sounding rockets can provide several minutes of a microgravity environment; but the goal of providing days, or even hours, for microgravity research has proved elusive. It was once hoped that the Space Shuttle could provide the desired environment; but such factors as manned activity, machine and structural vibrations, and thruster firings for orientation or reboost have resulted in acceleration levels generally unsatisfactory for the designed experiments. (Background excitations have been measured in the $10^{-3}g$ range.) In fact, the data from many experiments have been found unacceptable on account of the poor acceleration environment.

In view of the low frequencies of greatest concern (below about 10 Hz), the isolation problem is a largely unfamiliar one to vibration engineers; the requirement of a corner frequency of about 10^{-3} Hz is particularly vexing. Passive isolation is incapable of solving the isolation problem in this region; and even should a sufficiently soft spring be physically realizable, it could not isolate against direct disturbances to the payload. If the payload is tethered (e.g., for evacuation, power transmission, cooling, or material transport), a passive isolator cannot provide isolation below the corner frequency imposed by the umbilical stiffness.

An active isolator (such as a magnetic suspension system) that merely possesses a low positive stiffness fares no better in the presence of an umbilical, for the same reasons. And if the control system seeks to lower the corner frequency by adding negative stiffness to counteract the umbilical's stiffness, the system will (at best) possess almost no stability robustness. In the face of the usual umbilical nonlinearities and uncertainties, this situation is clearly unacceptable. At very low frequencies the rattlespace constraints become limiting,[1,2] so that any isolation system must have unit transmissibility in that region. In short, a microgravity isolator must be active, and it must be capable of dealing with the particular frequency-dependent complexities accompanying a tethered payload and a restrictive rattlespace.

The available acceleration data clearly point to a need for three-dimensional isolation.[3] Classical control design methods are not well suited for handling such problems; modern control methods provide a much more natural setting, opening up to the designer the power of the developing robust control synthesis and analysis tools, along with a variety of well-tested and progressive computational software packages.

The well-known H_2 synthesis [linear quadratic regulator (LQR) or linear quadratic Gaussian (LQG)] methodology is one such modern control method. It can readily provide an optimal feedback controller for a linearized plant (i.e., payload plus umbilical) subject either to no exogenous input (LQR case) or to white-noise disturbances only (LQG case). An optimal control found by H_2 synthesis minimizes a quadratic (energy-type) cost function, or performance index. Such a performance index is quite appropriate for the microgravity isolation problem, since it allows penalizing both the control energy required for isolation and the vibrational energy of the payload. Unfortunately, however, the application of this synthesis method to practical problems has been plagued by robustness difficulties. Granted, the standard LQR solution provides excellent robustness guarantees for the single-input, single-output (SISO) problem[4] and also yields guarantees (though less useful ones) for the multiple-input, multiple-output (MIMO) problem.[5] But the addition of a state observer to the controller (as is usually necessary for practical problems) removes these robustness guarantees.[6] This fundamental practical concern has led to a common skepticism regarding H_2 synthesis.

There exist extensions to H_2 synthesis, however, which can resolve the robustness issues. The disturbance-accommodation and frequency-weighting techniques contributed, respectively, by Johnson[7–10] and Gupta[11] have proved to be highly useful in this regard. In fact, they provide the fundamental additional tools needed for solving practical controller design problems. These two extensions lead to augmented state equations that still allow for problem solution by the familiar H_2 synthesis machinery. Recent investigations have examined the effect of the frequency-weighting extension on system robustness[12,13] and the dual relationship between frequency weighting and disturbance-accommodation.[14] Additional extensions have also been proposed.[15]

The utility of extended H_2 synthesis for the tethered microgravity vibration isolation problem has been clearly demonstrated by recent studies.[16,17] Extended H_2 synthesis has been used effectively to develop SISO and single-input, multiple-output (SIMO) controllers for a realistic one-dimensional microgravity vibration isolation problem, using a smart form of acceleration feedback. The resulting closed-loop system exhibited excellent stability and performance robustness guarantees, including a high degree of robustness to umbilical-parameter uncertainty.

The present paper will give a general framework for controller design by the extended H_2 synthesis method for the microgravity vibration isolation problem. Following a summary of the basic H_2 synthesis approach, the paper will describe how to incorporate the control and state frequency-weighting and input and output disturbance-accommodation extensions into the synthesis procedure. Control noise will also be included. General guidelines will be presented for effectively integrating these extensions into the design procedure.

The development below is specifically tailored to the microgravity isolation problem, but the mathematics are fully applicable to any problem that has the appropriate (very general) mathematical description. Four fundamental system models will be presented to aid the designer in visualizing the design effort. Only the synthesis procedure will be detailed here; the analysis techniques used for controller evaluation will be detailed in a later work.

Basic H_2 Synthesis Review

A generic microgravity vibration isolation system is depicted in Fig. 1. A payload, such as a microgravity science experiment, is acted upon by actuators (typically noncontacting) that are commanded by a control system. This control system uses measurements, such as payload positions and accelerations, to develop the control signals, typically currents or voltages. The objective of H_2 synthesis is to find a control signal that minimizes the weighted sum of the two-norms of the control energy and the states, subject to the linearized system equations of motion. This control signal will be found to be dependent only on the past accumulative measurement information, for a system excited only by zero-mean white Gaussian noise.

Specifically (and a bit more mathematically), to use H_2 synthesis the system equations of motion must first be linearized and expressed in the following (standard) state-space form:

$$\dot{x} = Ax + Bu + E_s w_s \tag{1a}$$

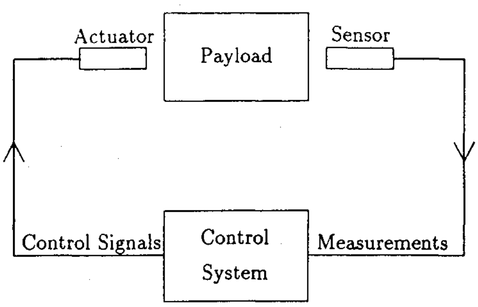

Fig. 1 Vibration isolation system.

$$y = Cx + Du \tag{1b}$$

$$z = y + E_n w_n \tag{1c}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{y}$ is the output vector, $\boldsymbol{z}$ is the measurement vector, $\boldsymbol{u}$ is the control vector, E_s and E_n are selection matrices, and $\boldsymbol{w}_s$ and $\boldsymbol{w}_n$ are process- and sensor-noise vectors, respectively. For the microgravity vibration isolation problem, the process noise models the disturbances acting on the payload, either directly (e.g., air currents, fluid flow, or experiment-mounted rotating machinery) or indirectly (i.e., through the umbilicals). The sensor noise models the electrical or mechanical noise that contaminates the state measurements. In general, not all states will be measurable (i.e., $\operatorname{rank} C \le \dim \boldsymbol{x}$).

Let the initial conditions on the state vector be $\boldsymbol{x}(0) = \boldsymbol{x}_0$ (although these initial conditions will not appear in the final control solution); let $\boldsymbol{x}_0$, $\boldsymbol{w}_s$, and $\boldsymbol{w}_n$ be independent and bounded (as is reasonable, since unbounded states and infinitely large noise are not physically possible), and let $\boldsymbol{x}_0$ be Gaussian, and $\boldsymbol{w}_n$ and $\boldsymbol{w}_s$ zero-mean white Gaussian, for technical reasons. The power of the process- and sensor-noise vectors can be expressed mathematically[18] by

$$\operatorname{cov}[\boldsymbol{w}_s(t), \boldsymbol{w}_s(\tau)] = V_1 \delta(t - \tau)$$

and

$$\operatorname{cov}[\boldsymbol{w}_n(t), \boldsymbol{w}_n(\tau)] = V_3 \delta(t - \tau) \tag{2}$$

Assume that $\{A, B\}$ and $\{A, E_s V_1^{1/2}\}$ are stabilizable, where $V_1 = V_1^{1/2} V_1^{1/2*}$ (the asterisk here means conjugate transpose), and that $\{C, A\}$ is detectable.[19] These requirements mean, respectively, that a stabilizing controller exists, and that the available measurements are sufficient for its implementation. Let V_1 and V_3 be positive semidefinite (PSD) and positive definite (PD), respectively, for reasons of solution existence. That is, there need not be any process noise, but there must be at least some noise in all measurement channels (as there will be) if an optimal-control solution is to exist.

The H_2 synthesis design method uses a quadratic performance index,

$$J = \mathcal{E}\left\langle [\boldsymbol{x}^T \quad \boldsymbol{u}^T] \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \begin{Bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \end{Bmatrix} \right\rangle \tag{3}$$

The weighting matrices, assigned by the designer, allow him to place a relative importance on the two-norm of each state (using W_1) and of the control (using W_3). W_2 allows him to assign cross weightings. (These cross weightings are not generally used for the basic H_2 synthesis problem, but they become important with some of the extensions.) W_1 is PSD and W_3 is PD,[18] again for reasons of solution existence. $\mathcal{E}$ is needed because the system is excited stochastically by $\boldsymbol{w}_s$.

Let an admissible control $\boldsymbol{u}(t)$ be one that depends only on the past accumulative observation data. That is, $\boldsymbol{u}(t)$ has the form

$$\boldsymbol{u}(t) = \boldsymbol{u}[t, \boldsymbol{Z}(t)] \tag{4a}$$

where

$$\boldsymbol{Z}(t) = \{\boldsymbol{z}(\tau), 0 \le \tau \le t\} \tag{4b}$$

(For more general conditions on admissibility, see Ref. 18.) The objective is to find an admissible control function $\boldsymbol{u}^*(t)$ that minimizes the cost J with respect to the set of admissible control functions $\boldsymbol{u}(t)$ subject to the dynamic constraint (1a–c). That is, the optimal-control solution must exist and be realizable, must minimize the cost functional specified by the designer, and must take into consideration the system equations of motion.

The solution is well known and is summarized as follows:

$$\boldsymbol{u}^*(t) = -K\tilde{\boldsymbol{x}}(t) \tag{5a}$$

where $\tilde{\boldsymbol{x}}$ is an estimate of $\boldsymbol{x}$ using a Luenberger observer[18] having observer gain matrix L, and

$$K = W_3^{-1}\left(B^T P + W_2^T\right) \tag{5b}$$

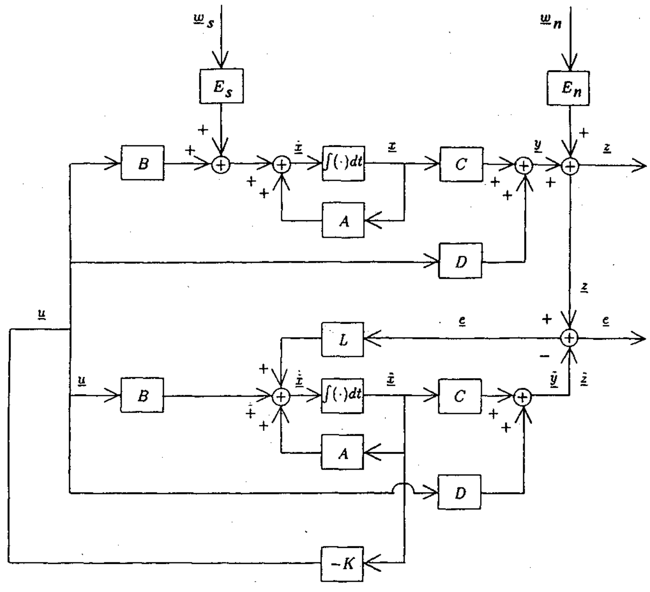

Fig. 2 Block diagram of system with H_2-optimal controller.

Here P is the unique PD solution to the algebraic Riccati equation (ARE)

$$PA + A^T P - (PB + W_2)W_3^{-1}(PB + W_2)^T + W_1 = O \tag{5c}$$

and

$$L = QC^T\left(E_n V_3 E_n^T\right)^{-1} \tag{5d}$$

where Q is the unique PD solution to the ARE

$$AQ + QA^T - QC^T\left(E_n V_3 E_n^T\right)^{-1} CQ + E_s V_1 E_s^T = O \tag{5e}$$

The solution P exists if $\{A, B\}$ is stabilizable and $\{C, A\}$ is detectable, or if the system is asymptotically stable; and Q exists if $\{A, E_s V_1^{1/2}\}$ is stabilizable and $\{C, A\}$ is detectable, or if the system is asymptotically stable.

Note that this control signal is developed by simply applying constant (negative) feedback gains to estimates of the system states. These estimates are themselves optimal in that they are the closest to the actual states, in terms of the expected value of the rms of the estimate error. They are produced in the controller from the control signal and the measurement vector, using constant observer gains. Figure 2 presents this standard optimal controller[20] in block-diagram form.

Extensions to H_2 Synthesis

Frequency Weighting

For the active microgravity vibration isolation problem, payload accelerations are of much greater concern at some frequencies than at others. Accelerations at higher frequencies can be handled passively; and very low-frequency accelerations correspond to such large displacements that they are essentially unisolable in view of practical rattlespace constraints. Rattlespace constraints also require that the relative displacements between space platform and payload be kept to a minimum at low frequencies. Control is needed at the lower frequencies, where the plant is best known and where the major isolation effort is desired. At higher frequencies, however, excessive control can excite unmodeled higher modes of the plant. Consequently it is desirable, through the performance index, to be able to penalize control strategies in a frequency dependent fashion. This can be achieved by weighting the states $\boldsymbol{x}$ and the control $\boldsymbol{u}$ in the cost rate functional so that the weightings are frequency dependent. The latter, extended H_2 synthesis problem will be seen to have the same form as the former one. It will have the simple difference that the various system matrices now will be augmented to take into account the additional pseudostates required by the frequency-weighting extension.

Let $\boldsymbol{x}$ be considered to be the input to filter $\mathcal{W}_1(s)$ of which $\bar{\boldsymbol{x}}$ is the output, and let $\mathcal{W}_1(s)$ have a state-space representation defined by $\{A_1, B_1, C_1, D_1\}$. That is,

$$\mathcal{W}_1(s) = C_1(sI - A_1)^{-1}B_1 + D_1 \tag{6}$$

Then

$$\dot{z}_1 = A_1 z_1 + B_1 \boldsymbol{x} \tag{7a}$$

$$\bar{\boldsymbol{x}} = C_1 z_1 + B_1 \boldsymbol{x} \tag{7b}$$

expresses $\bar{\boldsymbol{x}}$ in terms of $\boldsymbol{x}$, employing pseudostates z_1. Similarly, if $\boldsymbol{u}$ is considered to be the input to a filter $\mathcal{W}_3(s)$ of which $\bar{\boldsymbol{u}}$ is the output, and if $\mathcal{W}_3(s)$ has a state-space representation defined by $\{A_2, B_2, C_2, D_2\}$, then $\bar{\boldsymbol{u}}$ can be expressed in terms of $\boldsymbol{u}$, employing pseudostates z_2:

$$\dot{z}_2 = A_2 z_2 + B_2 \boldsymbol{u} \tag{8a}$$

$$\bar{\boldsymbol{u}} = C_2 z_2 + D_2 \boldsymbol{u} \tag{8b}$$

These frequency-weighted states ($\bar{\boldsymbol{x}}$) and controls ($\bar{\boldsymbol{u}}$) are now weighted (i.e., penalized) by constant weighting matrices W_1 and W_3, respectively. The resulting state equations and performance index are as follows:

$${}^1\dot{\boldsymbol{x}} = {}^1A\,{}^1\boldsymbol{x} + {}^1B\boldsymbol{u} + {}^1E_s\boldsymbol{w}_s \tag{9a}$$

$$\boldsymbol{y} = {}^1C\,{}^1\boldsymbol{x} + D\boldsymbol{u} \tag{9b}$$

$$\boldsymbol{z} = \boldsymbol{y} + E_n\boldsymbol{w}_n \tag{9c}$$

$${}^1J = \mathcal{E}\left\langle [{}^1\boldsymbol{x}^T \quad \boldsymbol{u}^T]\begin{bmatrix} {}^1W_1 & {}^1W_2 \\ {}^1W_2^T & {}^1W_3 \end{bmatrix}\begin{Bmatrix} {}^1\boldsymbol{x} \\ \boldsymbol{u} \end{Bmatrix}\right\rangle \tag{9d}$$

where

$${}^1\boldsymbol{x} = \begin{bmatrix} \bar{\boldsymbol{x}} \\ z_1 \\ z_2 \end{bmatrix} \tag{9e}$$

$${}^1A = \begin{bmatrix} A & O & O \\ B_1 & A_1 & O \\ O & O & A_2 \end{bmatrix} \tag{9f}$$

$${}^1B = \begin{bmatrix} B \\ O \\ B_2 \end{bmatrix} \tag{9g}$$

$${}^1C = [C \quad O \quad O] \tag{9h}$$

$${}^1E_s = \begin{bmatrix} E_s \\ O \\ O \end{bmatrix} \tag{9i}$$

$${}^1W_1 = \begin{bmatrix} D_1^T W_1 D_1 & D_1^T W_1 C_1 & O \\ C_1^T W_1 D_1 & C_1^T W_1 C_1 & O \\ O & O & C_2^T W_3 C_2 \end{bmatrix} \tag{9j}$$

$${}^1W_2 = \begin{bmatrix} O \\ O \\ C_2^T W_3 D_2 \end{bmatrix} \tag{9k}$$

$${}^1W_3 = \left[D_2^T W_3 D_2\right] \tag{9l}$$

The optimal control $\boldsymbol{u}(t)$ will now minimize the weighted sum of the two norm of the frequency-weighted control energy and states.

Input Disturbance Accommodation

In the basic H_2 problem it was assumed that the process noise $\boldsymbol{w}_s$ (i.e., the disturbance acting on the payload, whether directly, or indirectly via the umbilicals) is zero-mean white Gaussian. This, of course, will not generally be the case; the process noise will have some (known or unknown) nonwhite power spectrum. Let the process noise be modeled as $\boldsymbol{f}_s$, with power spectral density $S_f(\omega) = S_f^{1/2}(j\omega)S_f^{1/2}(j\omega)$. Defining $H_f(j\omega)$ by $S_f^{1/2}(j\omega)V_1^{1/2}$, one can consider $\boldsymbol{f}_S$ to be the output of a filter $H_f(s)$ excited by zero-mean white Gaussian noise $\boldsymbol{w}_s$ with power V_1 [i.e., $\mathrm{cov}[\boldsymbol{w}_s(t), \boldsymbol{w}_s(\tau)] = V_1\delta(t-\tau)$]. In state-space form,

$$\dot{\boldsymbol{\xi}}_1 = A_s\boldsymbol{\xi}_1 + B_s\boldsymbol{w}_s \tag{10a}$$

$$\boldsymbol{f}_s = C_s\boldsymbol{\xi}_1 + D_s\boldsymbol{w}_s \tag{10b}$$

so that

$$H_f(s) = C_s(sI - A_s)^{-1}B_s + D_s \tag{10c}$$

Incorporating these new pseudostates $\boldsymbol{\xi}_1$ into the state equations and performance index yields the further augmented H_2 synthesis problem given below:

$${}^2\dot{\boldsymbol{x}} = {}^2A\,{}^2\boldsymbol{x} + {}^2B\boldsymbol{u} + {}^2E_s\boldsymbol{w}_s \tag{11a}$$

$$\boldsymbol{y} = {}^2C\,{}^2\boldsymbol{x} + D\boldsymbol{u} \tag{11b}$$

$$\boldsymbol{z} = \boldsymbol{y} + E_n\boldsymbol{w}_n \tag{11c}$$

$${}^2J = [{}^2\boldsymbol{x}^T \quad \boldsymbol{u}^T]\begin{bmatrix} {}^2W_1 & {}^2W_2 \\ {}^2W_2^T & {}^2W_3 \end{bmatrix}\begin{Bmatrix} {}^2\boldsymbol{x} \\ \boldsymbol{u} \end{Bmatrix} = {}^1J \tag{11d}$$

where

$${}^2\boldsymbol{x} = \begin{Bmatrix} {}^1\boldsymbol{x} \\ \xi_1 \end{Bmatrix} \tag{11e}$$

$${}^2A = \begin{bmatrix} A & O & O & E_sC_s \\ B_1 & A_1 & O & O \\ O & O & A_2 & O \\ O & O & O & A_s \end{bmatrix} \tag{11f}$$

$${}^2B = \begin{bmatrix} B \\ O \\ B_2 \\ O \end{bmatrix} \tag{11g}$$

$${}^2C = [C \quad O \quad O \quad O] \tag{11h}$$

$${}^2E_s = \begin{bmatrix} E_sD_s \\ O \\ O \\ B_s \end{bmatrix} \tag{11i}$$

$${}^2W_1 = \begin{bmatrix} D_1^T W_1 D_1 & D_1^T W_1 C_1 & O & O \\ C_1^T W_1 D_1 & C_1^T W_1 C_1 & O & O \\ O & O & C_2^T W_3 C_2 & O \\ O & O & O & O \end{bmatrix} \tag{11j}$$

$${}^2W_2 = \begin{bmatrix} O \\ O \\ C_2^T W_3 D_2 \\ O \end{bmatrix} \tag{11k}$$

$${}^2W_3 = \left[D_2^T W_3 D_2\right] \tag{11l}$$

The optimal-control solution to this problem will minimize the frequency-weighted cost functional as before, with the plant now considered to be subject to the specified colored-noise disturbance.

In actual space applications the power spectrum of the process noise may not be known. Fortunately, orbiter spectral vibration information need not be available for disturbance accommodation to be used. In fact, the disturbances can be assumed to have whatever form the designer finds useful. Disturbance-accommodation filters are typically used only as tuning parameters, and not as restrictions on the types of disturbances for which the controller is effective. For example, if the designer wishes the controller not to respond to process noise above some frequency range, he might choose to model $H_f(s)$ as a low-pass filter. The resulting controller can be evaluated in terms of the associated closed-loop transfer function(s) between direct and indirect disturbances and payload vibration; and the attenuation of the various frequency components of the different disturbances can be easily predicted.

Output Disturbance Accommodation

The same procedure can be employed to incorporate colored sensor noise into the extended H_2 synthesis problem. No sensor will have the white-noise contamination assumed by the basic H_2 problem. And the designer may even find it useful to shape the sensor noise filter in some nonphysical way. For example, if payload acceleration measurements are known to be more accurate in one frequency range, and relative position measurements in another, he might choose his sensor noise filters appropriately to inform the observer of these facts. The resulting observer would tend to rely more heavily, in its state-reconstruction process, on the more accurate measurement(s) in a particular frequency range.

Let the sensor noise vector be f_n, with selection matrix E_n, where f_n is a stochastically modeled disturbance with power spectral density $S_n(j\omega) = S_n^{1/2}(j\omega)S_n^{(1/2)*}(j\omega)$. As with the input disturbance f_s, f_n can be considered to be the output of a filter $H_n(s)$ excited by zero-mean white Gaussian noise w_n with power V_3 [i.e., $\mathrm{cov}[w_n(t), w_n(\tau)] = V_3\delta(t-\tau)$]. In state-space form,

$$\dot{\xi}_2 = A_n\xi_2 + B_n w_n \tag{12a}$$

$$f_n = C_n\xi_2 + D_n w_n \tag{12b}$$

so that

$$H_n(s) = C_n(sI - A_n)^{-1}B_n + D_n \tag{12c}$$

For the extended H_2 synthesis problem with state and control frequency weighting (pseudostates z_1 and z_2, respectively), and with input and output disturbance accommodation (pseudostates ξ_1 and ξ_2, respectively), the augmented state equations and the performance index are as follows:

$$^3\dot{x} = {}^3A\,{}^3x + {}^3Bu + {}^3E_s\,{}^3w_s \tag{13a}$$

$$z = {}^3C\,{}^3x + {}^3Du + {}^3E_n\,{}^3w_n \tag{13b}$$

$$^3J = \mathcal{E}\left\langle [{}^3x^T \quad u^T]\begin{bmatrix} {}^3W_1 & {}^3W_2 \\ {}^3W_2^T & {}^3W_3 \end{bmatrix}\begin{Bmatrix} {}^3x \\ u \end{Bmatrix}\right\rangle \tag{13c}$$

where

$$^3x = \begin{Bmatrix} x \\ z_1 \\ z_2 \\ \xi_1 \\ \xi_2 \end{Bmatrix} \tag{13d}$$

$$^3A = \begin{Bmatrix} A & O & O & E_sC_s & O \\ B_1 & A_1 & O & O & O \\ O & O & A_2 & O & O \\ O & O & O & A_s & O \\ O & O & O & O & A_n \end{Bmatrix} \tag{13e}$$

$$^3B = \begin{Bmatrix} B \\ O \\ B_2 \\ O \\ O \end{Bmatrix} \tag{13f}$$

$$^3C = [C \quad O \quad O \quad O \quad E_nC_n] \tag{13g}$$

$$^3D = D \tag{13h}$$

$$^3E_s = \begin{Bmatrix} E_sD_s & O \\ O & O \\ O & O \\ B_s & O \\ O & B_n \end{Bmatrix} \tag{13i}$$

$$^3w_s = \begin{Bmatrix} w_s \\ w_n \end{Bmatrix} \tag{13j}$$

$$^3w_n = \{w_n\} \tag{13k}$$

$$^3E_n = [E_nD_n] \tag{13l}$$

$$^3W_1 = \begin{Bmatrix} D_1^TW_1D_1 & D_1^TW_1C_1 & O & O & O \\ C_1^TW_1D_1 & C_1^TW_1C_1 & O & O & O \\ O & O & C_2^TW_3C_2 & O & O \\ O & O & O & O & O \\ O & O & O & O & O \end{Bmatrix} \tag{13m}$$

$$^3W_2 = \begin{Bmatrix} O \\ O \\ C_2^TW_3D_2 \\ O \\ O \end{Bmatrix} \tag{13n}$$

$$^3W_3 = \left[D_2^TW_3D_2\right] \tag{13o}$$

$$^3V_1 = \begin{bmatrix} V_1 & O \\ O & V_3 \end{bmatrix} \tag{13p}$$

is the autocorrelation matrix for ^3w_s,

$$^3V_2 = \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} \tag{13q}$$

is the cross-correlation matrix between $\{{w_s \atop w_n}\}$ and w_n, and

$$^3V_3 = V_3 \tag{13r}$$

is the autocorrelation matrix for w_n.

Control Noise

The active microgravity vibration isolation system must perform well even when the actual system dynamics are less than perfectly described by the system model. One way to improve the isolation system's robustness to parameter changes at the control inputs (i.e., to uncertainties in the actuator or plant model) is to add a process noise input (w_c) to the model's control signal.[21] Recall that the H_2 synthesis machinery seeks to minimize the rms of the observation error. It does this by finding an observer gain matrix L that will optimally trade off the measurement uncertainties against the plant model uncertainties in the state reconstruction process. Control noise will reduce the observer's confidence in the plant model, so that the observer will trust its measurement data more and its plant model less. The resulting gain matrix L will sacrifice a degree of observation quality for improved observer robustness to plant model inaccuracies. The controller gain matrix K will be unaffected.

Under these circumstances the state equations of motion, unaugmented by frequency weighting or disturbance accommodation, become

$$\dot{x} = Ax + B(u + w_c) + E_s f_s \tag{14a}$$

$$y = Cx + D(u + w_c) \tag{14b}$$

$$z = y + E_n f_n \tag{14c}$$

where f_s and f_n can be represented by filters in state-space form with white-noise inputs, as noted previously. Assume no cross correlation between w_c and w_s, or between w_c and w_n; and let

$$\text{cov}[w_c(t), w_c(\tau)] = V_c\delta(t - \tau) \tag{15}$$

Using now the left-hand superscript 4 to indicate the appropriate state-space augmentation, the system equations change as follows:

$$^4x = {}^3x \tag{16a}$$

$$^4A = {}^3A, \quad {}^4B = {}^3B, \quad {}^4C = {}^3C, \quad {}^4D = {}^3D \tag{16b}$$

$$^4W_1 = {}^3W_1, \quad {}^4W_2 = {}^3W_2, \quad {}^4W_3 = {}^3W_3 \tag{16c}$$

$$^4E_s = \begin{bmatrix} B & E_sD_s & O \\ O & O & O \\ B_2 & O & O \\ O & B_s & O \\ O & O & B_n \end{bmatrix} \tag{16d}$$

$$^4w_s = \begin{Bmatrix} w_c \\ w_s \\ w_n \end{Bmatrix} \tag{16e}$$

$$^4w_n = \begin{Bmatrix} w_c \\ w_n \end{Bmatrix} \tag{16f}$$

$$^4E_n = [D \quad E_nD_n] \tag{16g}$$

$$^4V_1 = \begin{bmatrix} V_c & O & O \\ O & V_1 & O \\ O & O & V_3 \end{bmatrix} \tag{16h}$$

$$^4V_2 = \begin{bmatrix} V_c & O \\ O & V_2 \\ O & V_3 \end{bmatrix} \tag{16i}$$

$$^4V_3 = \begin{bmatrix} V_c & O \\ O & V_3 \end{bmatrix} \tag{16j}$$

The basic tools are now in place for practical design of a microgravity vibration isolation system by extended H_2 synthesis.

System Modeling

The H_2 synthesis problem is actually a twofold design problem: the designer must determine a regulator gain matrix as well as an observer gain matrix; these matrices are used together to make up the optimal controller. The full augmented state vector must be used for the regulator subdesign problem, but the pseudostates z_1 and z_2 (which arise from frequency weighting) need not be reconstructed by the observer. They can simply be developed by passing the reconstructed state vector x and the control vector u through the appropriate frequency-weighting filters. Consequently the observer subdesign problem can (though it need not) be one of reduced order. It is helpful, then, to have different mathematical models for conceptualizing these two subproblems. These models will also differ, in general, from the basic plant model, which depicts the actual linearized plant (i.e., the system without the controller) in state-space form. This model will not include the frequency-weighting and disturbance-accommodation augmentations of the former. In addition to these three conceptual models of the system, there is a fourth model, which more properly falls under the category of analysis, but should be kept in mind during the synthesis procedure. This nominal analysis model depicts the linearized and unaugmented plant with the synthesized controller attached. It is used, with various modifications, to analyze closed-loop system performance.

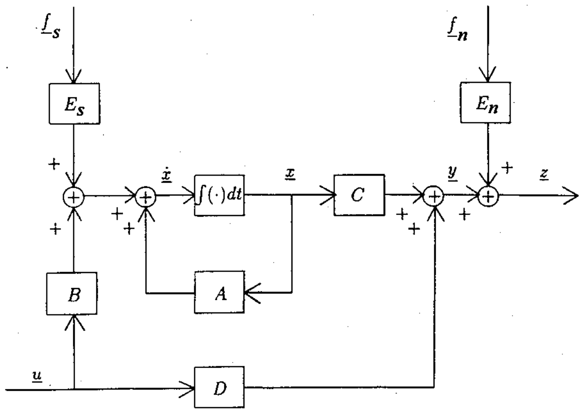

Fig. 3 Basic plant model.

Basic Plant Model

The basic plant model (shown schematically in Fig. 3) simply presents the linearized differential equations of motion in a state-space form useful to the H_2 synthesis machinery. Such a representation is given below:

$$\dot{x} = Ax + Bu + E_s f_s \tag{17a}$$

$$y = Cx + Du \tag{17b}$$

$$z = y + E_n f_n \tag{17c}$$

No performance index is needed at this stage, since it is the frequency-weighted states and control that will be weighted relative to each other for the actual controller synthesis. For the microgravity vibration isolation problem a useful choice for the state vector would include relative displacements, relative velocities, and accelerations. Weighting an acceleration more heavily in a frequency range would correspond roughly to a demand to increase the associated effective mass (or inertia) of the system. A similar correspondence can be drawn between relative-displacement weighting and the effective relative stiffness, and between relative-velocity weighting and the effective relative damping.

Regulator Synthesis Model

The regulator synthesis model adds frequency weighting and disturbance-accommodation weighting filters to the basic plant model, and is the model actually used by the extended H_2 synthesis machinery in designing the regulator. Figure 4 portrays this model in block-diagram form. Note that it has a similar form to the basic plant model, but that now augmented $A, B, C, E_s, E_n, W_1, W_2$, and W_3 matrices are used, as indicated by the left-hand superscripts. These matrices were defined previously Eqs. (16b–16g). Also note that the white-noise vector w_s replaces f_s, since the spectral information on the process noise power is now contained in the matrices ^4A and ^4E_s.

The regulator synthesis model shows the system as viewed by the extended H_2 machinery in determining the matrix ^4P. This matrix is the unique positive definite solution to the following ARE:

$$^4P\,^4A + {}^4A^T\,^4P - \left(^4P\,^4B + {}^4W_2\right) \times {}^4W_3^{-1}\left(^4P\,^4B + {}^4W_2\right)^T + {}^4W_1 = O \tag{18}$$

Here ^4P is used to find the regulator feedback gains. This model is typically used only in determining the regulator gains ^4K. It is

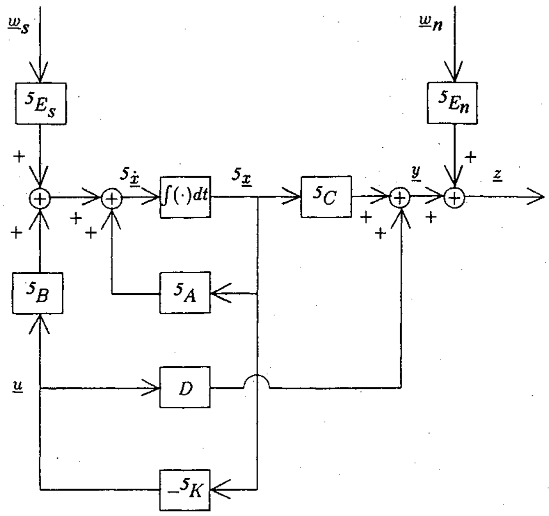

Fig. 4 Regulator synthesis model.

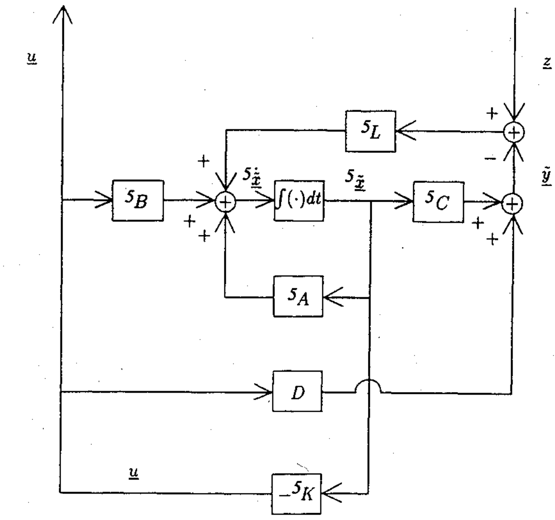

Fig. 5 Observer synthesis model, observing all states and pseudostates.

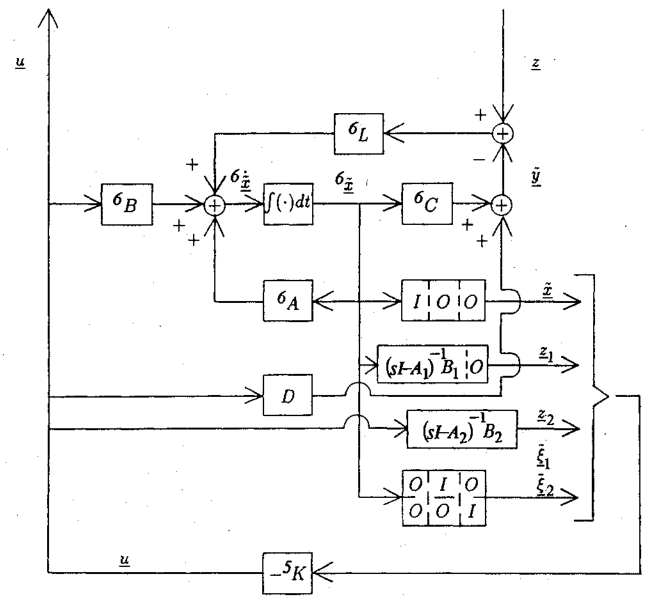

Fig. 6 Reduced observer synthesis model.

not generally used for the design of observer gains $^{(\cdot)}L$ (although it could be), since that design problem can be reduced to one of lower order, as noted before.

Observer Synthesis Model

The observer uses the measurement vector z (e.g., measured relative positions and accelerations) and the control vector u as inputs to produce observations of the state vector. (In general an observer is needed to estimate the unmeasurable system states. Only rarely will all system states actually be available for measurement, and never will the disturbance-accommodation pseudostates actually be capable of measurement.) Kalman–Bucy filter design uses knowledge of the process-noise and sensor-noise covariance matrices (and, if necessary, cross-correlation matrices) to produce optimal observer gains $^{(\cdot)}L$, in the sense of optimality previously discussed. If the complete augmented state vector ^4x is to be observed, then an appropriate observer synthesis model would be as depicted in Fig. 5. The matrix ^4L is found from the equation

$$^4L = \left(^4Q\,^4C^T + {}^4E_s\,^4V_2\,^4E_n^T\right)\left(^4E_n\,^4V_3\,^4E_n^T\right)^{-1} \tag{19a}$$

where ^4Q is the unique PD solution to the following ARE:

$$^4\tilde{A}\,^4Q + {}^4Q\,^4\tilde{A}^T - {}^4Q\,^4C^T\left(^4E_n\,^4V_3\,^4E_n^T\right)^{-1}\,^4C\,^4Q + {}^4E_s\,^4\tilde{V}_1\,^4E_s^T = O \tag{19b}$$

and where

$$^4\tilde{A} = {}^4A - \left(^4E_s\,^4V_2\,^4E_n^T\right)\left(^4E_n\,^4V_3\,^4E_n^T\right)^{-1}\,^4C \tag{19c}$$

$$^4\tilde{V}_1 = {}^4V_1 - {}^4V_2\,^4E_n^T\left(^4E_n\,^4V_3\,^4E_n^T\right)^{-1}\,^4E_n\,^4V_2^T \tag{19d}$$

and $^4V_1$, $^4V_2$, $^4V_3$, and ^4E_s are as defined previously.

Equations (19b–19d) reduce to the form

$$^4A\,^4Q + {}^4Q\,^4A^T - \left(^4Q\,^4C^T + {}^4E_s\,^4V_2\,^4E_n^T\right)\left(^4E_n\,^4V_3\,^4E_n^T\right) + \left(^4Q\,^4C^T + {}^4E_s\,^4V_2^T\,^4E_n^T\right) + {}^4E_s\,^4V_1\,^4E_3^T = O \tag{19e}$$

However, as noted previously, the observer does not need to reconstruct the frequency-weighting pseudostates z_1 and z_2. This fact will permit an observer of smaller dimension. In this case, Fig. 6 will be an appropriate observer synthesis model, with pertinent matrices defined as follows. ^5L will be found from the equation

$$^5L = \left(^5Q\,^5C^T + {}^5E_s\,^5V_2\,^5E_n^T\right)\left(^5E_n\,^5V_3\,^5E_n^T\right)^{-1} \tag{20a}$$

where ^5Q is the unique PD solution to the ARE

$$^5\tilde{A}\,^5Q + {}^5Q\,^5\tilde{A}^T - {}^5Q\,^5C^T\left(^5E_n\,^5V_3\,^5E_n^T\right)^{-1}\,^5C\,^5Q + {}^5E_s\,^5\tilde{V}_1\,^5E_s^T = O \tag{20b}$$

and where

$$^5\tilde{A} = {}^5A - \left(^5E_s\,^5V_2\,^5E_n^T\right)\left(^5E_n\,^5V_3\,^5E_n^T\right)^{-1}\,^5C \tag{20c}$$

$$^5\tilde{V}_1 = {}^5V_1 - {}^5V_2\,^5E_n^T\left(^5E_n\,^5V_3\,^5E_n^T\right)^{-1}\,^5E_n\,^5V_2^T \tag{20d}$$

$$^5V_1 = {}^4V_1 \tag{20e}$$

$$^5V_2 = {}^4V_2 \tag{20f}$$

$$^5V_3 = {}^4V_3 \tag{20g}$$

$$^5E_s = \begin{bmatrix} B & E_sD_s & O \\ O & B_s & O \\ O & O & B_n \end{bmatrix} \tag{20h}$$

$$^5A = \begin{bmatrix} A & E_sC_s & O \\ O & A_s & O \\ O & O & A_n \end{bmatrix} \tag{20i}$$

$$^5C = [C \quad O \quad E_nC_n] \tag{20j}$$

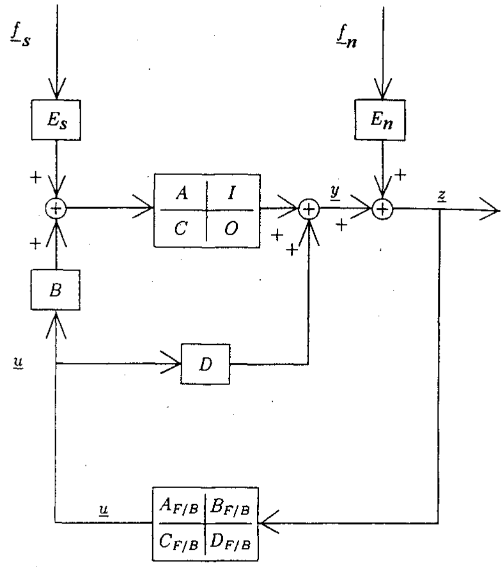

Fig. 7 Nominal analysis model.

Equations (20b–20d) reduce to the form

$$
{}^5A\,{}^5Q + {}^5Q\,{}^5A^T - \left({}^5Q\,{}^5C^T + {}^5E_s\,{}^5V_2\right)
$$

$$
\times\,{}^5V_3^{-1}\left({}^5Q\,{}^5C^T + {}^5E_s\,{}^5V_2\right)^T + {}^5E_s\,{}^5V_1\,{}^5E_s^T = O \qquad (20\mathrm{k})
$$

Nominal Analysis Model

Once the active vibration-isolation controller has been designed, it can usually be reduced in size by modal truncation and/or balance-and-truncate.[22] Then the closed-loop system can be evaluated, with the controller applied to the actual plant.

Letting the state-space system $\{A_{\mathrm{FB}}, B_{\mathrm{FB}}, C_{\mathrm{FB}}, D_{\mathrm{FB}}\}$ represent the feedback controller, a nominal analysis model can be portrayed as in Fig. 7. Checks on nominal stability can be made by simple eigenvalue checks of this closed-loop system. Nominal performance can also be conducted readily using this model. With the appropriate placement of complex Δ blocks to represent system uncertainties, one can also evaluate the system for robust stability and robust performance guarantees, using the powerful methods of μ analysis.[23]

Design Philosophy for the Microgravity Isolation Problem

With the synthesis framework now in place, as presented above, the designer must choose a reasonable strategy to use his H_2 synthesis tools with skill. He must determine what states to use, what frequency-weighting and disturbance-accommodation filters (if any) to employ, the relative weightings of the resultant frequency-weighted states and control, and the relative weightings of the various noise vectors in his system model. The designer must also decide which measurements to make, whether to use the full or the reduced observer synthesis model, and whether to conduct the regulator- and observer-gain subproblems in sequence or in combination.

One primary goal in controller design should be simplicity. The construction of the controller will be easiest, and its operation fastest, if its complexity (i.e., the number of controller states) is kept to a minimum. To accomplish this aim, the authors recommend that the engineer seek to design the controller by starting with basic (unextended) H_2 synthesis and adding complexity one layer at a time. For example, he might first determine if basic LQG is adequate, and then add appropriate frequency weighting and disturbance accommodation step by step, evaluating after each addition whether or not the design is acceptable. If not, the next layer of complexity could be added, based on the current design inadequacies. Once an adequate design has been found, it is recommended that the controller order (i.e., number of states) be reduced by using modal reduction and/or balance-and-truncate. This step-up, step-down philosophy should keep controller complexity to a minimum.

A second fundamental goal should be intuitiveness. Unless the problem is posed in such a way as to employ the designer's intuition, he will find it very difficult, especially with a three-dimensional problem, to proceed with any degree of speed. The single most important step toward an intuitive problem is the proper choice of plant states. For the microgravity vibration isolation problem, the authors believe that a reasonably physical choice is payload relative position, payload relative velocity, and payload acceleration. A heavier weighting on payload relative position (in the cost-functional matrix W_1), for example, signals the H_2 machinery to attempt to increase system stiffness. Similar analogs exist for the other two suggested states, as noted before. And at least two of these states are readily measurable for microgravity systems. Such state choices, then, allow the designer to assign his weightings with a degree of physical feel, so that extended H_2 synthesis becomes more of a craftsman's design tool than a black box for use in a time-consuming trial-and-error approach.

The designer must also decide whether or not to conduct the regulator- and observer-gain design problems independently. The well-known separation principle guarantees that for a perfectly known system the regulator gains K and the observer gains L can be designed independently. One approach, then, would be first to design the regulator to meet the design goals, and then to design the observer to produce a state-vector estimate that is accurate enough over a sufficient bandwidth. The frequency-weighting and disturbance-accommodation extensions, however, affect the state observations in such a manner that "accurate enough" and "sufficient" are quite difficult terms to define. The closed-loop system must be analyzed as a whole for this purpose. The existence of an observer bandwidth can also be used to enhance overall system performance, so that a full-state-feedback system with inadequate performance can actually perform quite well when the observer is added. Since the system stability and performance robustness must ultimately be evaluated for the total closed-loop system, it is recommended that the entire controller (i.e., the observer plus regulator) be designed as a unit, rather than in parts.

For the microgravity vibration isolation problem, studies to date indicate that H_2 synthesis extensions are necessary if one is to produce a practical control.[16] This being so, there are certain frequency weightings that are very reasonable choices to use. At very low frequencies, indirect disturbances (i.e., orbiter positional deviations from a perfectly elliptical orbit) will be much larger than rattlespace constraints will allow. In the low-frequency region, then, the payload relative displacement should be weighted heavily, and the payload acceleration, lightly. These weighting choices could reasonably be expected to call for a controller producing unit transmissibility between orbiter and payload at low frequencies. In the intermediate frequency range, where payload acceleration is of most concern, that state should be weighted heavily. At higher frequencies, where the plant model is not well known, high control weightings and low state weightings should be used to call for reduced control.

Certain disturbance-accommodation filters, as well, will be appropriate for the problem, while others will be inadvisable. From a physical perspective, a more massive experiment would be less susceptible either to direct or to indirect disturbances. One could expect, then, that an input disturbance filter which models a large direct disturbance would call for a controller tending to make the system seem more massive (electronically). On the other hand, an indirect disturbance alone (i.e., one acting through the umbilical) could be attenuated effectively either by a greater system effective mass or by a reduced system effective stiffness. The latter means of disturbance attenuation is ineffective for direct disturbances. It also tends to reduce the stability robustness of the system. Hence, the designer should be wary of having too large an indirect disturbance model.

Output disturbance accommodation and control noise should be included in the system model only if necessary. Research to date does not indicate that either is needed for microgravity vibration isolator design. Again, the goal is to achieve a satisfactory controller that is as simple as possible.

Observer design involves the numerical solution of an ARE. An ARE involving matrices of smaller dimension will be less susceptible to the numerical difficulties that sometimes attend such solution procedures. It is preferable, then, to use the reduced rather than the full observer synthesis model.

Concluding Remarks

Active vibration isolation of microgravity science experiments is a three-dimensional, MIMO design problem requiring sophisticated design and analysis tools. Modern control methods provide the most natural setting for handling this problem; and with a suitable choice of states, modern control design can be conducted in a fairly intuitive fashion. The H_2 synthesis approach can be extended, using frequency-weighting and disturbance-accommodation techniques, to give the designer great flexibility in building a suitable controller. Implementation of these extensions involves a straightforward augmentation of various system matrices, so that the ARE-based solution methods of LQG synthesis can be readily applied. Extended H_2 synthesis provides the necessary tools for the design of a robust isolation system. This paper has provided a general framework for using extended H_2 synthesis to design the controller for such a system.

In addition to the basic plant model, there are three complementary system models that are of use in conceptualizing the synthesis problem. Observer synthesis requires fewer pseudostates than regulator synthesis, so two respective system models are needed to reflect this difference. The controller model is developed by combining the observer and regulator models, followed by reduction of the controller dimensionality. Attachment of this controller to the basic plant model produces an analysis model that can be used, with μ-analysis methods, to evaluate the closed-loop system in terms of its stability and performance robustness.

A general design philosophy has also been suggested for applying the extended H_2 synthesis machinery to the particular design problem at hand. It was recommended that relative positions, relative velocities, and payload accelerations be the states of choice used for the system model, since these permit design-filter selection with a great degree of physical intuition. It was also suggested that the entire controller (observer plus regulator) be designed as a unit, rather than in two parts, since the observer bandwidth can help the regulator do its job of disturbance attenuation. Finally, it was recommended that disturbance-accommodation and frequency-weighting design filters be chosen that call for increased effective stiffness at low frequencies (to cause the payload to track the orbiter), increased mass at intermediate frequencies (to improve disturbance attenuation without sacrificing stability robustness), and controller turnoff at higher frequencies, where higher, unmodeled system modes tend to dominate.

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